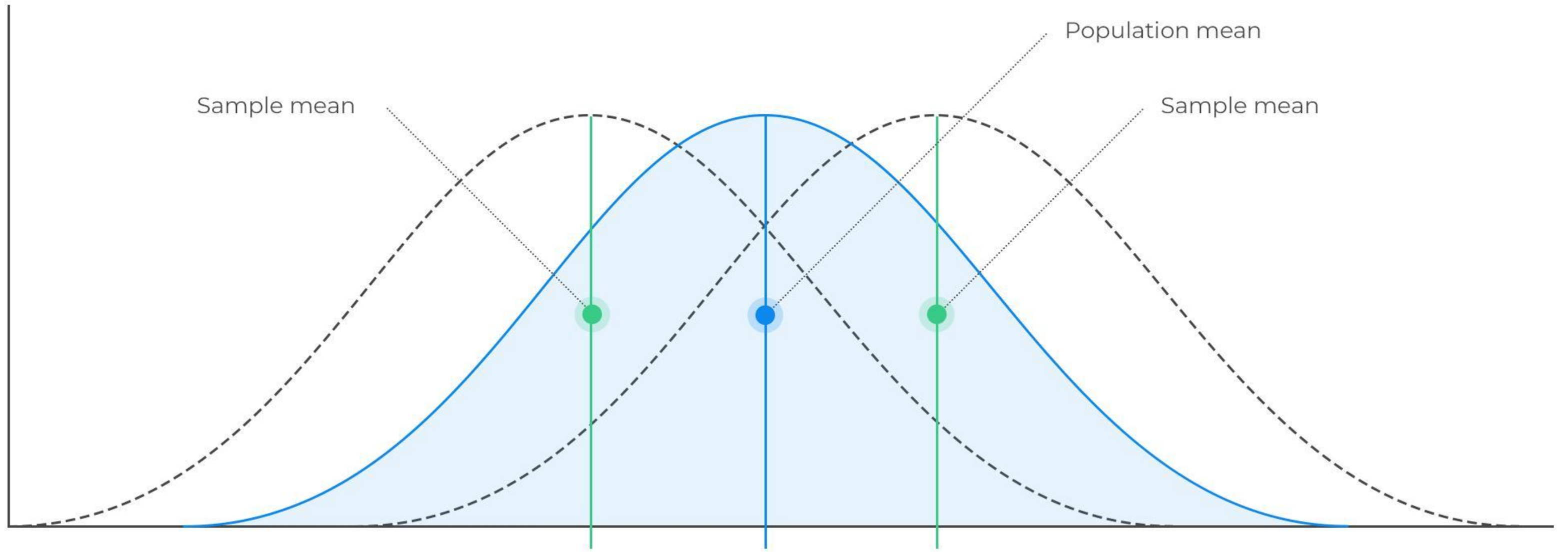
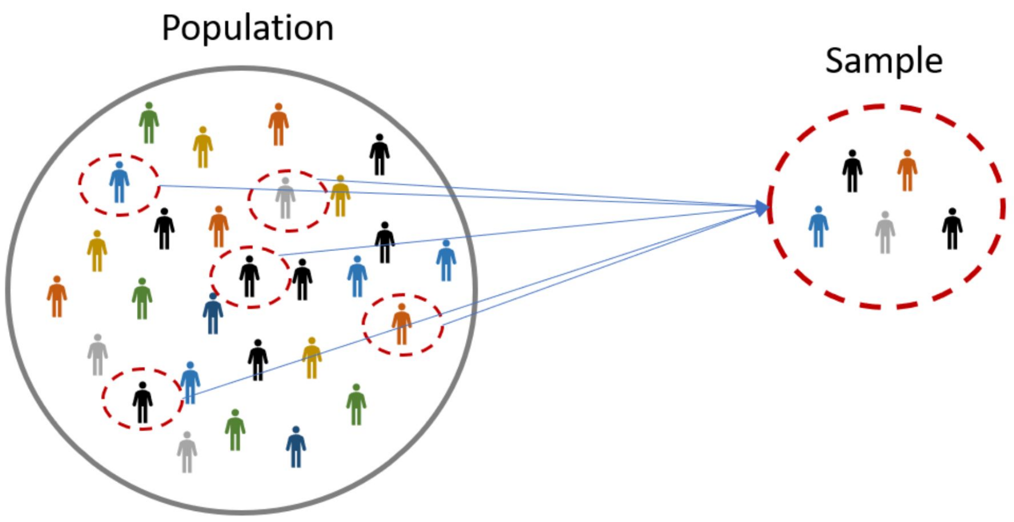


Sampling Distributions

Samples and population

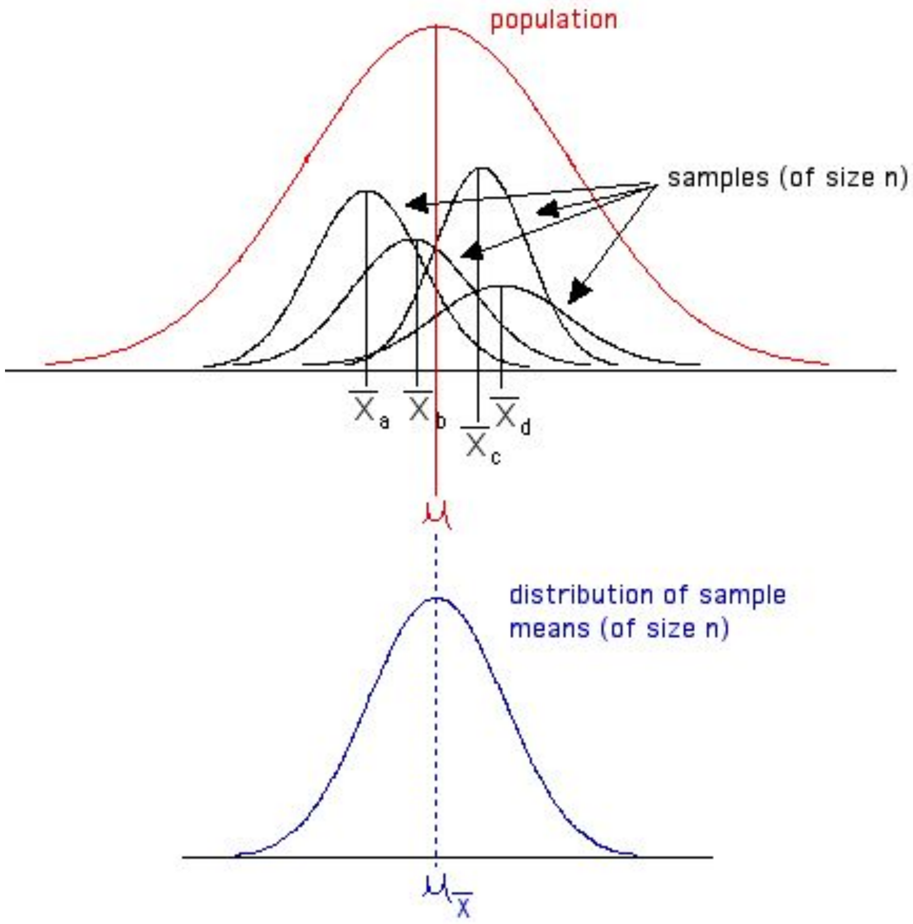
- The location of a score in a sample or in a population can be represented with a *z*-score
- Researchers typically want to study entire samples rather than single scores
  - Sample provides estimate of the population
  - Tests involve transforming sample mean to a *z*-score



Distribution of sample means

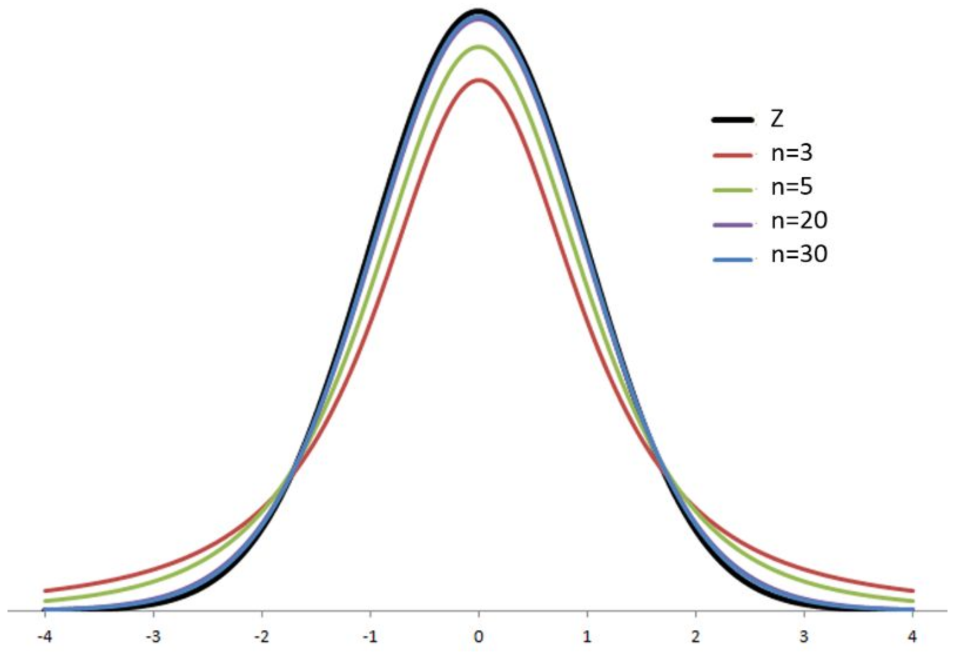
- Samples differ from each other
  - Given a random sample it is unlikely that sample means would always be the same

- The distribution of sample means = the collection of sample means for all the possible random samples of a particular size (*n*) that can be obtained from a population

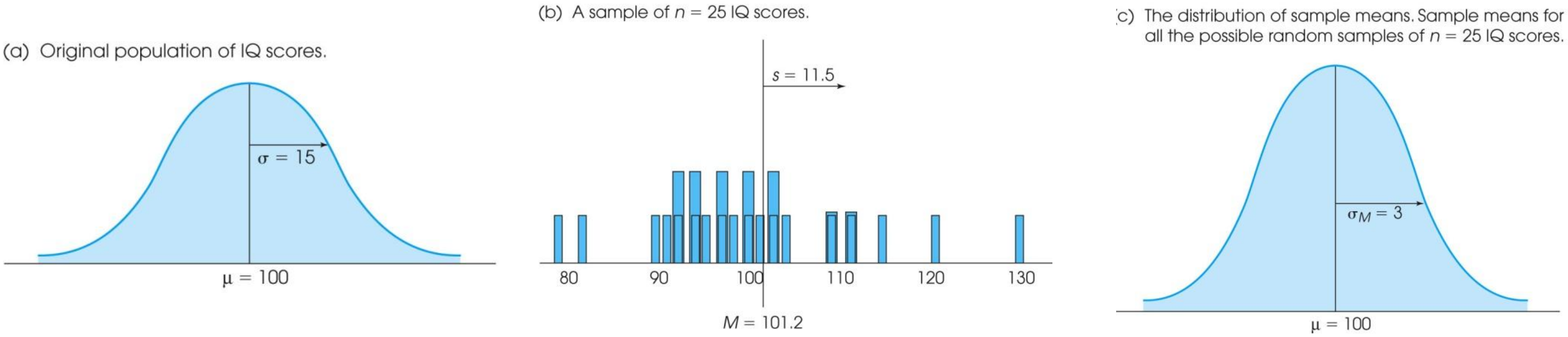


Sampling distribution

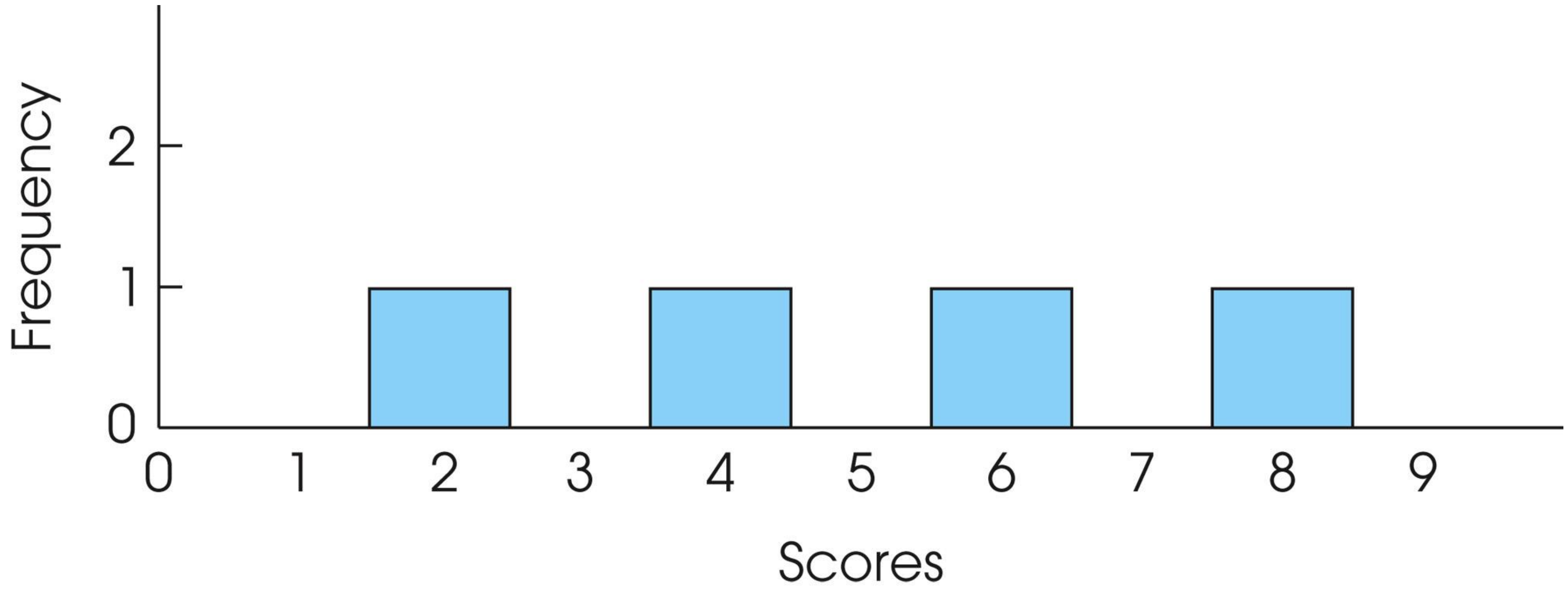
- Distribution of Sample Means is a distribution of sample means obtained from a population
  - Special kind of population -> a sampling distribution



Distribution of scores vs. Sample means

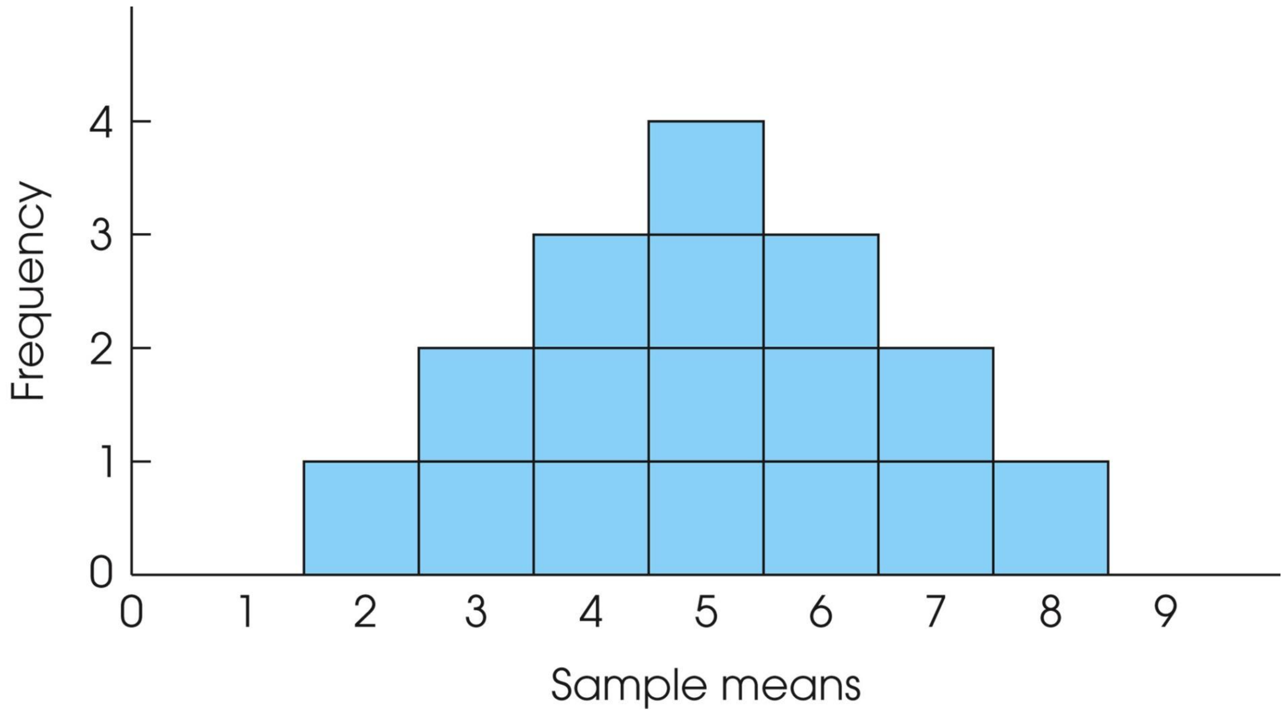


Population frequency distribution histogram



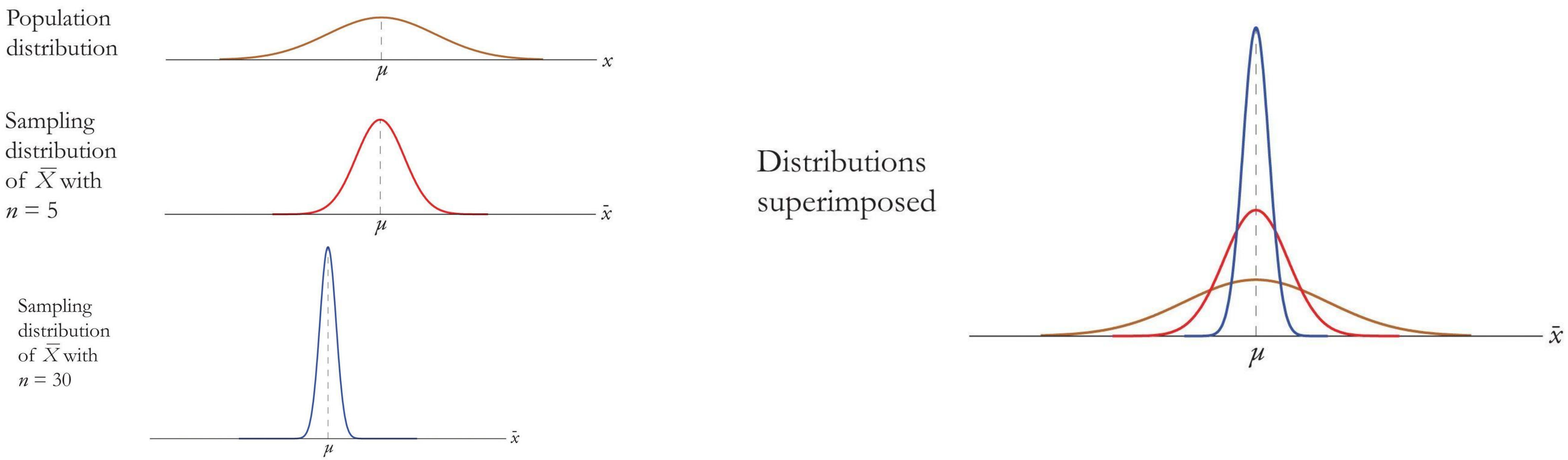
Distribution of sample means (n = 2)

SAMPLE	SCORES		SAMPLE MEAN
	FIRST	SECOND	(M)
1	2	2	2
2	2	4	3
3	2	6	4
4	2	8	5
5	4	2	3
6	4	4	4
7	4	6	5
8	4	8	6
9	6	2	4
10	6	4	5
11	6	6	6
12	6	8	7
13	8	2	5
14	8	4	6
15	8	6	7
16	8	8	8



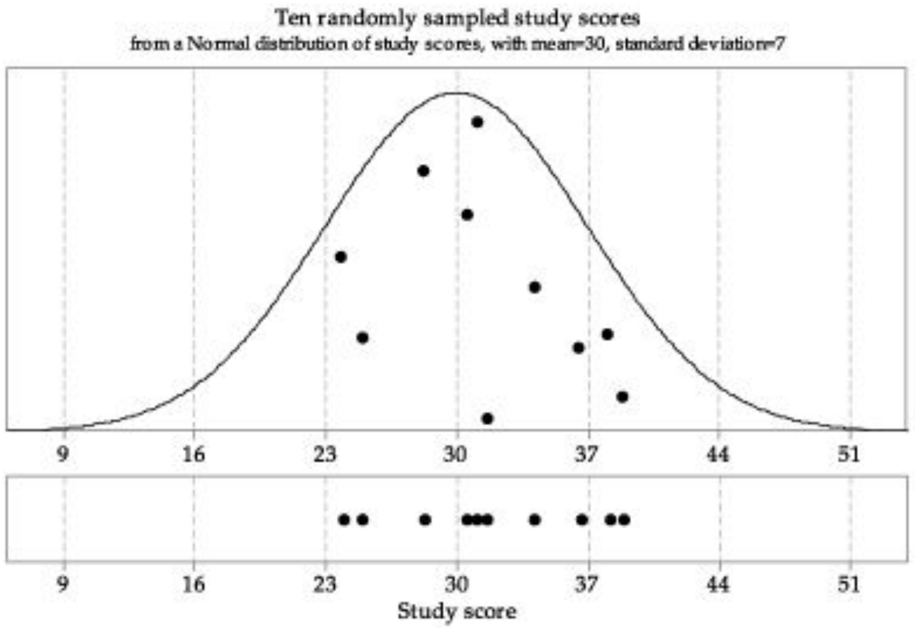
Important characteristics of distributions of sample means

- The sample means pile up around the population mean
- The distribution of sample means is approximately normal in shape

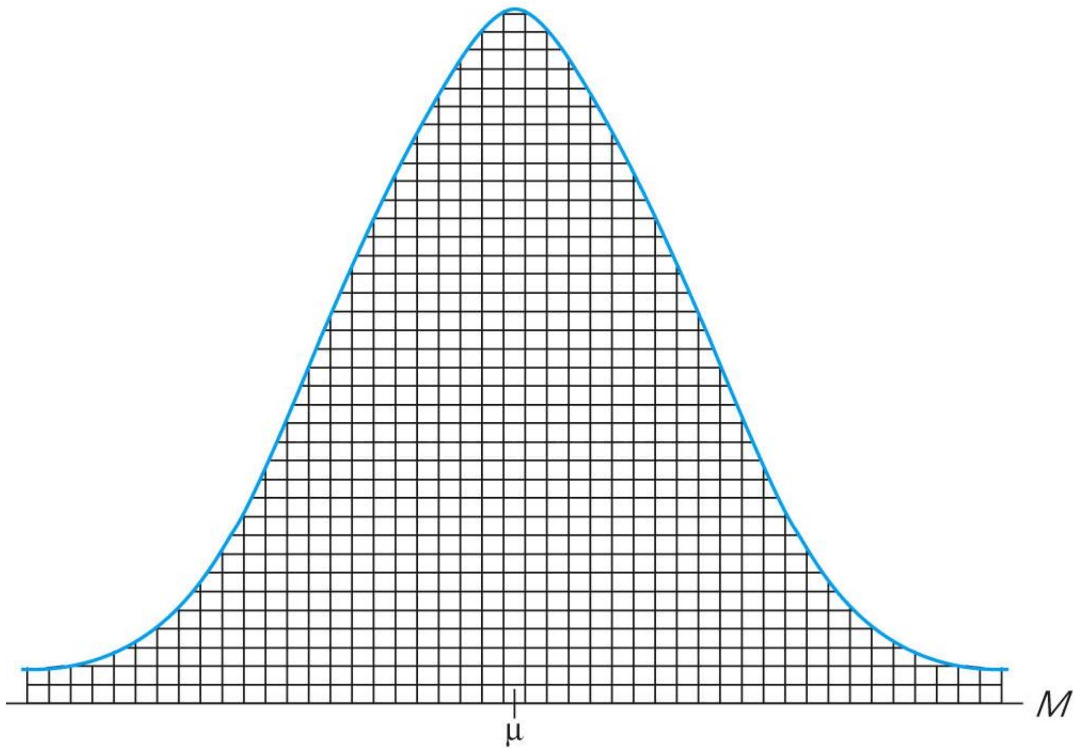


Shape of the distribution of sample means

- The distribution of sample means is almost perfectly normal in either of two conditions
  1. The population from which the samples are selected is a normal distribution or
  2. The number of scores (*n*) in each sample is relatively large—at least 30

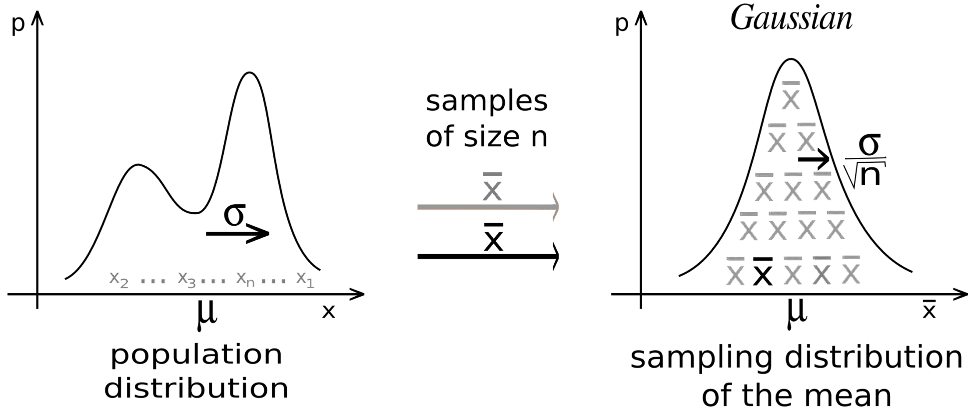


Example of typical distribution of sample means

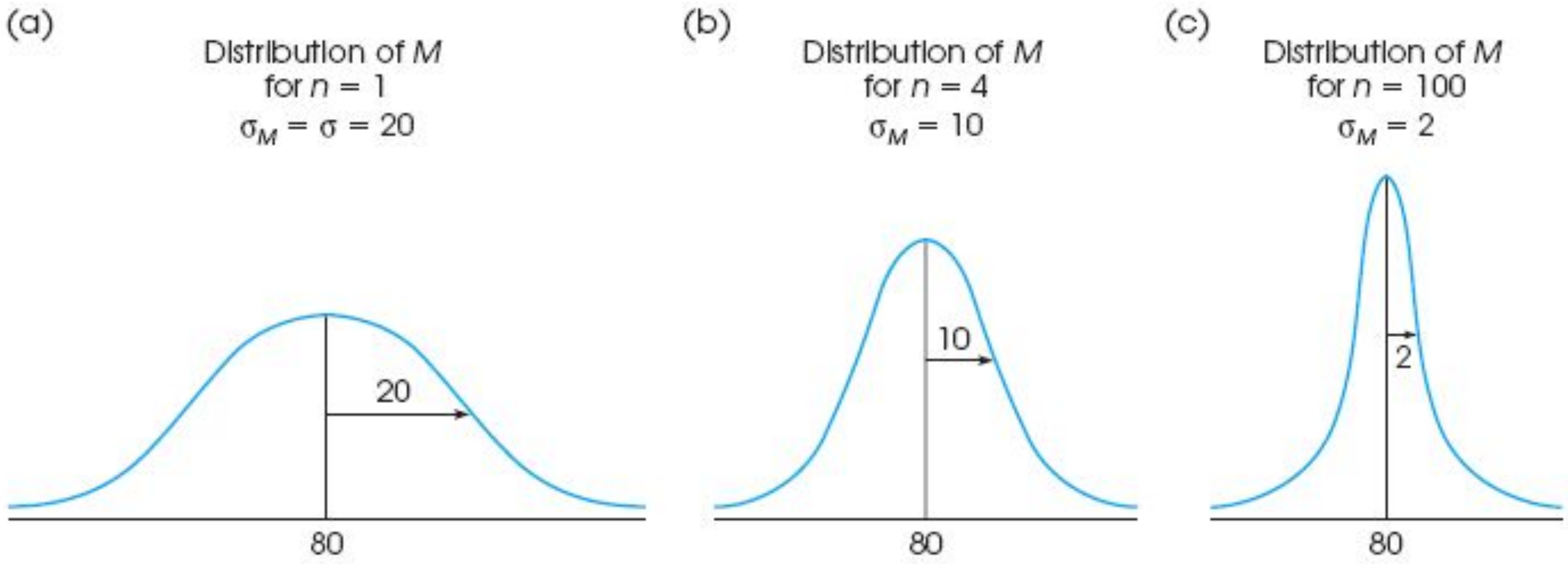


Central Limit Theorem

- Applies to any population with mean μ and standard deviation σ
- Distribution of sample means approaches a normal distribution as *n* approaches infinity
- Distribution of sample means for samples of size *n* will have a mean of μ_M
- Distribution of sample means for samples of size *n* will have a standard deviation = $\sigma / \sqrt{n}$



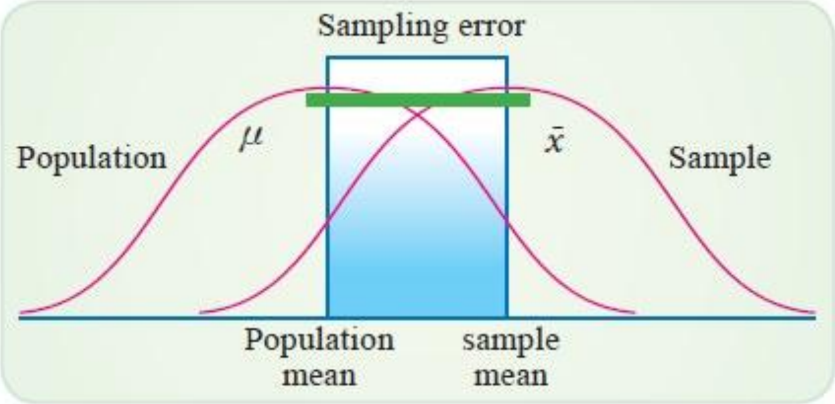
Distribution of sample means when n = 1, 4, and 100



Sampling error

- Because samples differ from one another, errors can occur

- Sampling error is the discrepancy between a sample statistic and its corresponding population parameter
  - Amount of sampling error varies across samples
  - The variability of sampling error is measured by the standard error of the mean



Expected value of M

- Mean of the distribution of sample means is μ_M and has a value equal to the mean of the population of scores, μ
  - ***expected value*** of *M*
- *M* is an *unbiased statistic* because μ_M , the expected value of the distribution of sample means is the value of the population mean, μ

Standard error of M

- Standard deviation = Variability of a distribution of <u>*scores*</u>
- <u>*Standard error of M*</u> ($\boldsymbol{\sigma_M}$) = Variability of a distribution of sample <u>*means*</u>
  - Measured by the standard deviation of the sample means
  - Provides a measure of how much distance is expected on average between M and μ
  - Standard error = $\sigma / \sqrt{n}$

Standard error magnitude

- **Law of large numbers:** the larger the sample size, the more probable it is that the sample mean will be close to the population mean
  - N $\uparrow$, M = μ, because SE $\downarrow$

- **Population variance:** The smaller the variance in the population, the more probable it is that the sample mean will be close to the population mean
  - σ $\downarrow$, then M = μ

$$SE = \frac{\sigma}{\sqrt{n}}$$

Example – Calculating SE

$$SE = \frac{\sigma}{\sqrt{n}}$$

A population has a standard deviation of $\sigma = 20$.

A) If a single score is selected from this population, how close, on average, would you expect this score to be to the population mean?

B) If a sample of n = 4 scores is selected from the population, how close, on average, would you expect the sample mean to be the population mean?

Example – Calculating *SE* ANSWERS $SE = \frac{\sigma}{\sqrt{n}}$

A population has a standard deviation of σ = 20.

A) If a single score is selected from this population, how close, on average, would you expect this score to be to the population mean?

B) If a sample of n = 4 scores is selected from the population, how close, on average, would you expect the sample mean to be the population mean?

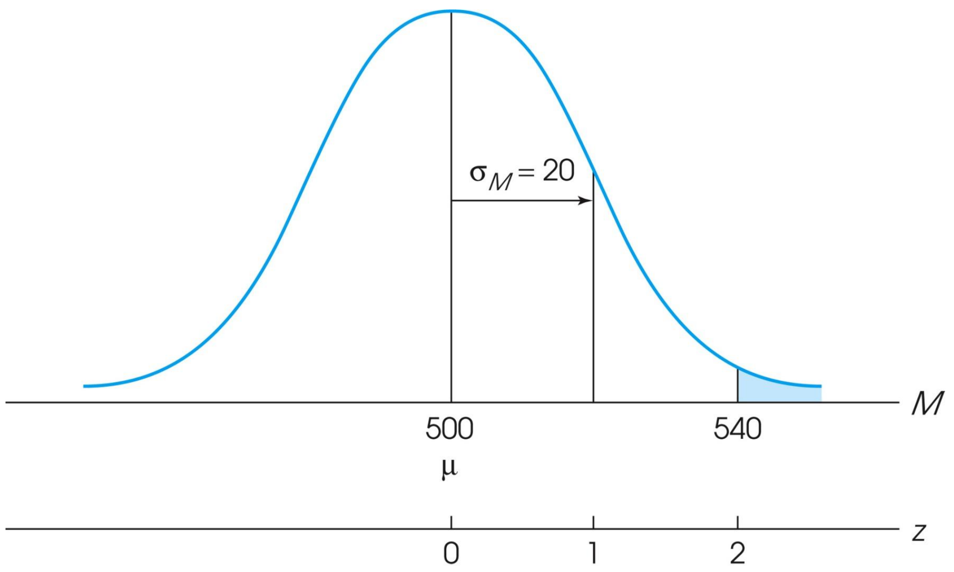
A) The standard deviation, σ = 20, measures the standard distance between a score and the mean

B) $SE = \frac{20}{\sqrt{4}} = \frac{20}{2}$ = 10 points

Probability and the distribution of sample means

- Primary use of the distribution of sample means is to find the probability associated with any particular sample (sample mean)
- Proportions of the normal curve are used to represent probabilities
- A *z*-score for the sample mean is computed

Distribution of sample means for n = 25



Z	.00
2.0	.97725

A z-score for sample means

- Sign tells whether the location is above (+) or below (-) the mean
- Number tells the distance between the location and the mean in standard deviation (standard error) units
- *z*-formula:

$$z = \frac{M - \mu}{\sigma_M}$$

Example – Calculating z-scores

A population has μ = 80 and σ = 12. Find the z-score corresponding to each of the sample means:

A) M = 84 for a sample of n = 9 scores

B) M = 74 for a sample of n = 16 scores

Formulas:

$$SE = \frac{\sigma}{\sqrt{n}} \qquad z = \frac{M - \mu}{\sigma_M}$$

Example – Calculating z-scores ANSWERS

A population has μ = 80 and σ = 12. Find the z-score corresponding to each of the sample means:

A) M = 84 for a sample of n = 9 scores

- $$SE = \frac{12}{\sqrt{9}} = \frac{12}{3} = 4\,; z = \frac{84-80}{4} = \frac{4}{4} = 1.00$$

B) M = 74 for a sample of n = 16 scores

$$SE = \frac{12}{\sqrt{16}} = \frac{12}{4} = 3\,; z = \frac{74-80}{3} = \frac{-6}{3} = -2.00$$

Learning Check 1

1. A population has $\mu = 60$ with $\sigma = 5$; the distribution of sample means for samples of size $n = 4$ selected from this population would have an expected value of ______

A) 5

B) 60

C) 30

D) 15

Learning Check 1 - Answer ☑

1. A population has $\mu = 60$ with $\sigma = 5$; the distribution of sample means for samples of size $n = 4$ selected from this population would have an expected value of ______

A) 5

B) 60

C) 30

D) 15

Learning Check 2

2. Decide if each of the following statements is True or False.

A) The shape of a distribution of sample means is always normal

B) As sample size increases, the value of the standard error decreases

Learning Check 2 - Answer ☑

2. Decide if each of the following statements is True or False.

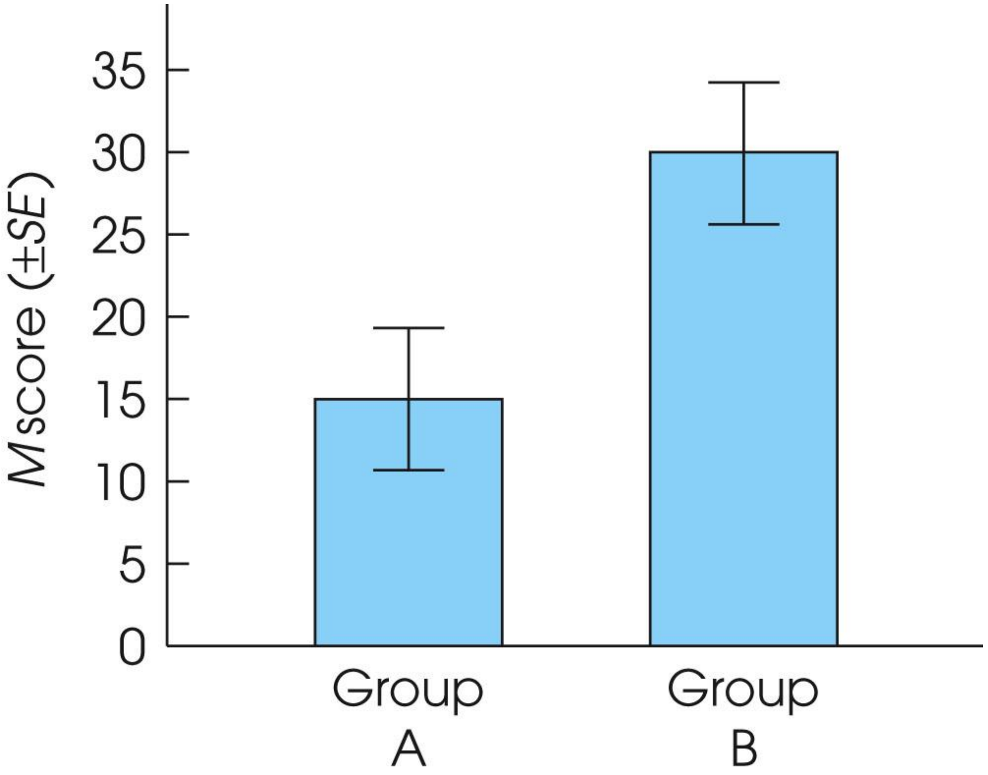
A) False - The shape is normal *only* if the population is normal or $n \geq 30$

B) True - Sample size is in the denominator of the equation so as n grows larger, standard error decreases

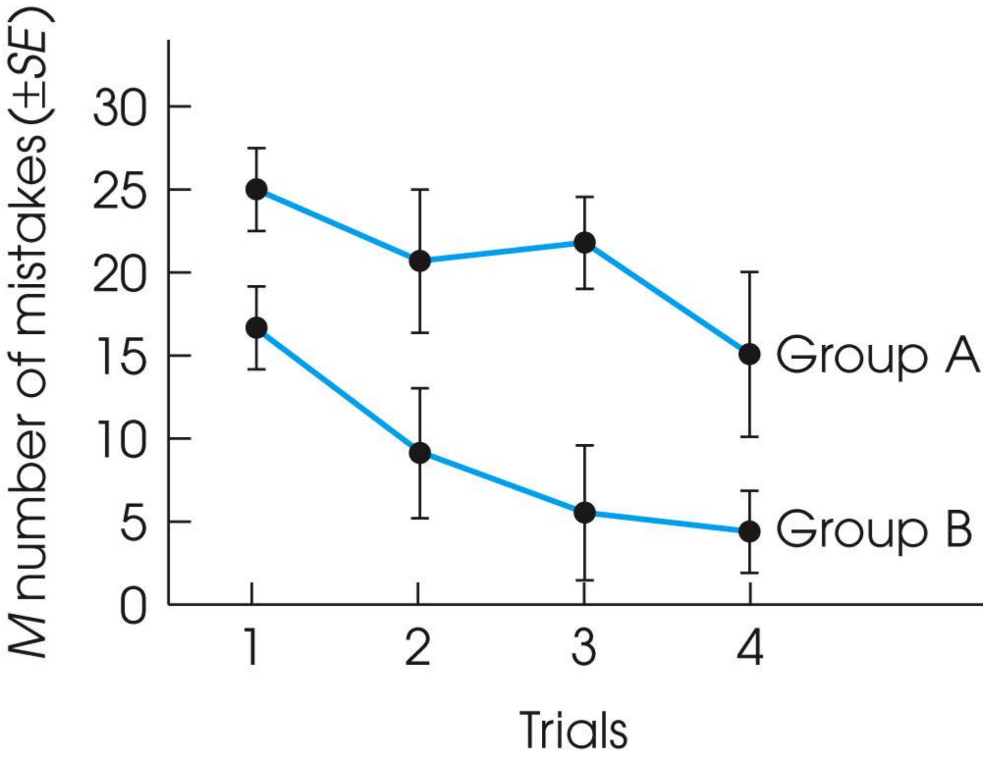
In the literature

- Journals vary in how they refer to the standard error but frequently use:
  - *SE*
  - *SEM*
- Often reported in a table along with *n* and *M* for the different groups in the experiment
- May also be added to a graph

SE in the literature



Mean (±1 *SE*) in bar chart



Mean (±1 *SE*) in line graph

Looking ahead to inferential statistics

- Inferential statistics use sample data to draw general conclusions about populations
  - Sample information is not a perfectly accurate reflection of its population (sampling error)
  - Differences between sample and population introduce uncertainty into inferential processes
- Statistical techniques use probabilities to draw inferences from sample data

Learning Check 3

1. A random sample of $n = 16$ scores is obtained from a population with $\mu = 50$ and $\sigma = 16$. If the sample mean is $M = 58$, the z-score corresponding to the sample mean is _____?

   A) $z = 1.00$

   B) $z = 2.00$

   C) $z = 4.00$

   D) Cannot determine

Learning Check 3 - Answer ☑

1. A random sample of n = 16 scores is obtained from a population with μ = 50 and σ = 16. If the sample mean is M = 58, the z-score corresponding to the sample mean is _____?

   A) z = 1.00

   B) z = 2.00

   C) z = 4.00

   D) Cannot determine

Learning Check 4

2. Decide if each of the following statements is True or False.

A) A sample mean with $z = 3.00$ is a fairly typical, representative sample

B) The mean of the sample is always equal to the population mean

Learning Check 4 - Answer

2. Decide if each of the following statements is True or False.

A) False - A *z*-score of 3.00 is an extreme, or unlikely, *z*-score

B) False - Individual samples will vary from the population mean

Learning Objectives

By the end of this lecture, you should be able to:

- Define distribution of sampling means
- Describe distribution by shape, expected value, and standard error
- Describe location of sample mean *M* by *z*-score