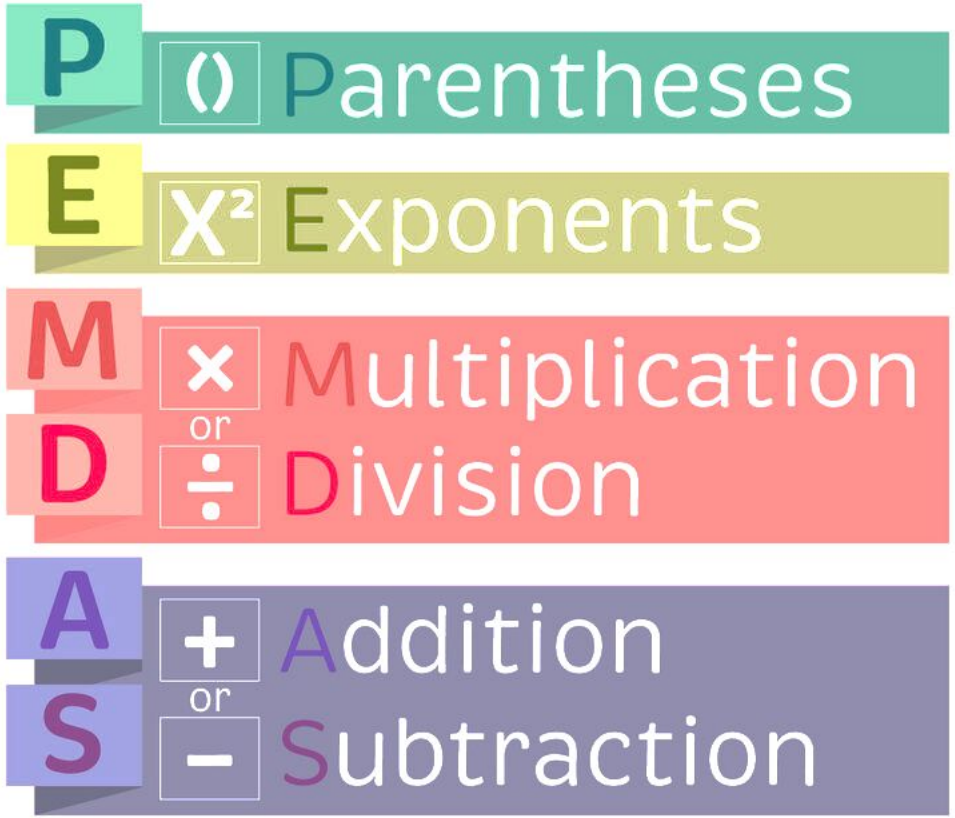
Applying the order of operations for algebra and statistics

Review - Order of operations



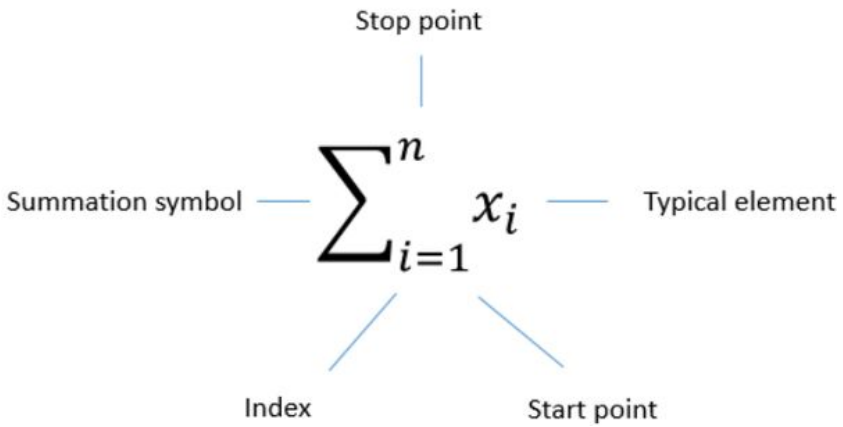
What mathematical operations are required in statistics?

- Addition
- Subtraction
- Division
- Multiplication
- Radicals (i.e., square roots)
- Exponents
- Summations (Σ)
- Factorials (!)

DOING THESE IN ORDER IS IMPORTANT!

Order of operations is a useful tool

- Knowing how to complete operations in order will help you accurately compute summations and factorials
- These operations are necessary in statistics


Stop point
Summation symbol
$\sum_{i=1}^{n} x_i$
Typical element
Index
Start point

Summations

What are summations?

- This is the upper case Greek letter sigma: Σ

- A sigma tells us that we need to sum (i.e., add) a series of numbers

Summations

- For example, four children are comparing how many pieces of candy they have

ID	Child	Pieces of Candy
1	Marty	9
2	Harold	8
3	Eugenia	10
4	Kevi	8

We could say that:

$$x_1 = 9$$
$$x_2 = 8$$
$$x_3 = 10$$
$$x_4 = 8$$

Basic summation

- If we wanted to know how many total pieces of candy the group of children had, we could add the four numbers
- The notation for this is: $\Sigma\, x_i$
- So, for this example, $\Sigma\, x_i = 9 + 8 + 10 + 8 = 35$
- Combined, the children have 35 pieces of candy

Sums of squares

- In statistics, some equations include the sum of all of the squared values (i.e., square each item, then add)

- The notation is: $\Sigma\, x_i^2$ or $\Sigma(x_i^2)$

- In our example, $\Sigma\, x_i^2 = 9^2 + 8^2 + 10^2 + 8^2$

$= 81 + 64 + 100 + 64$

$= 309$

Squares for previously summed numbers

- Sometimes we want to square a series of numbers that have already been added
- The notation for this is: $(\Sigma x_i)^2$
- In our example, $(\Sigma x_i)^2 = (9 + 8 + 10 + 8)^2 = 35^2 = 1225$
- Note that $\Sigma\, x_i^2$ and $(\Sigma x_i)^2$ are different

Video - Summations as used in introductory statistics

Summation – Example 1

Solve the following problem, with x values of 5, 6, 4, 3, and 5.

Solve:

$\sum x_i$

Summation – Example 1 Answer

Solve the following problem, with x values of 5, 6, 4, 3, and 5.

Answer:

$$\Sigma\, x_i = 5 + 6 + 4 + 3 + 5$$

$$= 23$$

Summation – Example 2

Solve the following problem, with x values of 5, 6, 4, 3, and 5.

Solve:

$\sum x_i^2$

Summation – Example 2 Answer

Solve the following problem, with x values of 5, 6, 4, 3, and 5.

Answer:

$$\sum x_i^2$$

$$\sum x_i^2 = 5^2 + 6^2 + 4^2 + 3^2 + 5^2$$

$$= 25 + 36 + 16 + 9 + 25$$

$$= 111$$

Summation – Example 3

Solve the following problem, with x values of 5, 6, 4, 3, and 5.

Solve:

$(\Sigma x_i)^2$

Summation – Example 3 Answer

Solve the following problem, with x values of 5, 6, 4, 3, and 5.

Answer:

$$(\Sigma x_i)^2 = (5 + 6 + 4 + 3 + 5)^2$$

- $$= 23^2$$

$$= 529$$

$$n! = n \times (n-1) \times (n-2) \times ... \times 1$$

$1! = 1$
$2! = 2 \times 1 = 2$
$3! = 3 \times 2 \times 1 = 6$
$4! = 4 \times 3 \times 2 \times 1 = 24$
$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

Factorials

Factorials

- Factorials are symbolized by exclamation points (!)
- A factorial is a mathematical operation in which you multiply the given number by all of the positive whole numbers less than it
- In other words: $n! = n \times (n-1) \times 2 \ \ldots \times 1$

Note that $0! = 1$

Factorials

For example:

- "Four factorial" $= 4! = 4 \times 3 \times 2 \times 1 = 24$
- "Six factorial" $= 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

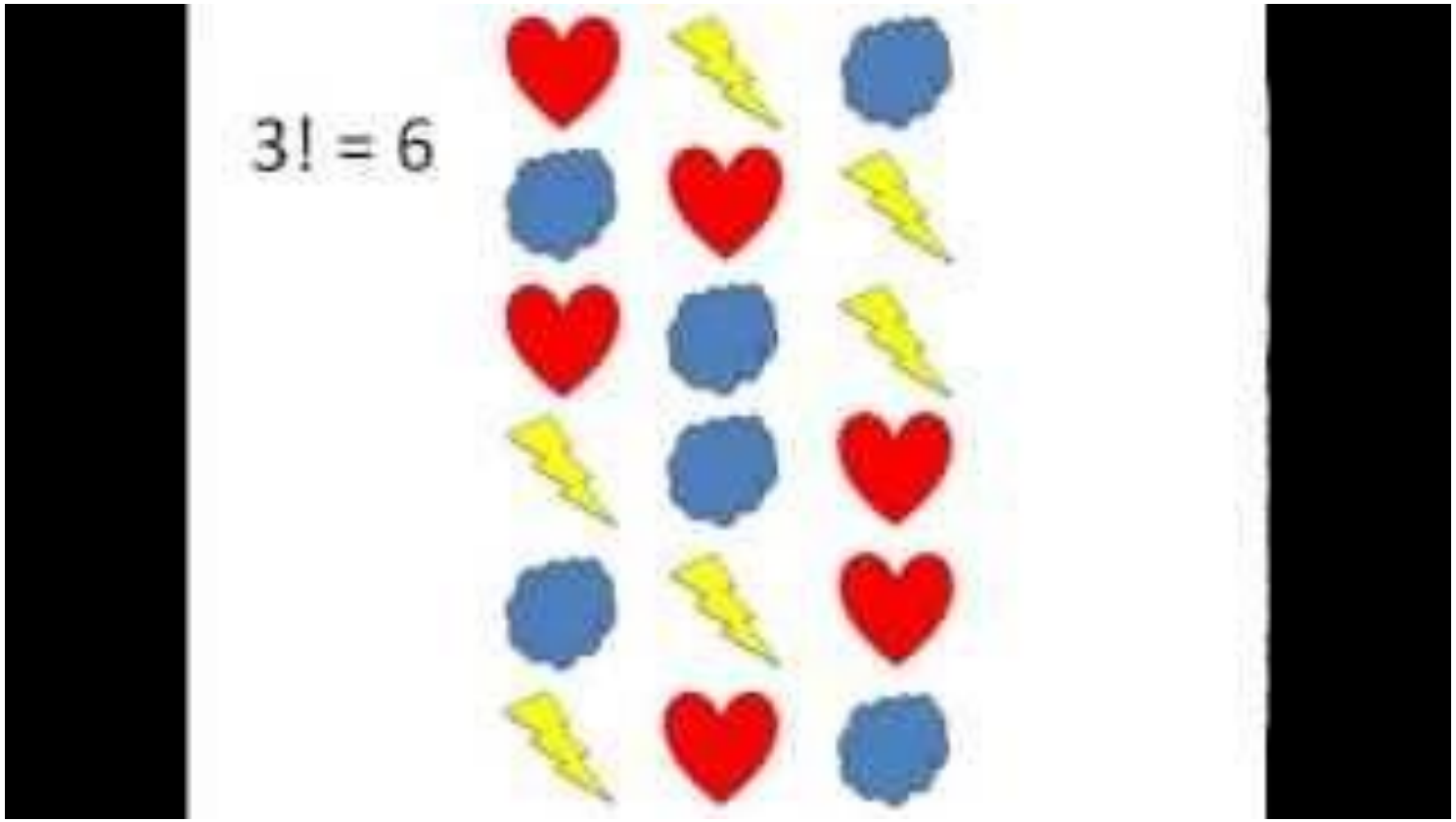
Factorials

- When we discuss probability distributions in statistics, we will see a formula that involves dividing factorials

- For example: $\frac{3!}{2!} = \frac{3 \times 2 \times 1}{2 \times 1} = 3$

- Or: $\frac{6!}{2!(6-2)!} = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{(2 \times 1)(4 \times 3 \times 2 \times 1)} = \frac{6 \times 5}{2} = \frac{30}{2} = 15$

Video – Use of factorials in statistics



Factorials – Examples

Solve the following factorials.

Factorials – Example 1

Solve:

$7!$

Factorials – Example 1 Answer

Answer:

$$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5{,}040$$

Factorials – Example 2

Solve:

$$\frac{4!}{2!}$$

Factorials – Example 2 Answer

Answer:

$$\frac{4!}{2!} = \frac{4 \times 3 \times 2 \times 1}{2 \times 1} = 12$$

Learning objectives

By the end of this review, you should be able to:

- Calculate summations
- Compute factorials